Specimen MA - C1

1)

$$\sum_{r=1}^{n} k = nk \quad , \quad \sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{20}(5+2r) = \sum_{r=1}^{20} 5 + 2\sum_{r=1}^{20} r$$

$$= (20 \times 5) + 2\left[\tfrac{1}{2}20(20+1)\right]$$

$$= 100 + 420$$

$$= 520$$

OR

$$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$$

$$S = 5 + 2r \quad \therefore a = 7, \; d = 2$$

$$S_{20} = \frac{1}{2}20\left[2(7) + (20-1)2\right]$$

$$= 10 \times (14 + 38)$$

$$= 520$$

2)

$$\int (5x + 3\sqrt{x})\, dx = 5\int x\, dx + 3\int x^{\frac{1}{2}} dx$$

$$= 5\frac{x^2}{2} + 3\frac{2x^{\frac{3}{2}}}{3} + C$$

$$= \frac{5x^2}{2} + 2x^{\frac{3}{2}} + C$$

3)

a)

$$\sqrt{80} = \sqrt{16} \times \sqrt{5}$$
$$= 4\sqrt{5}$$

b)

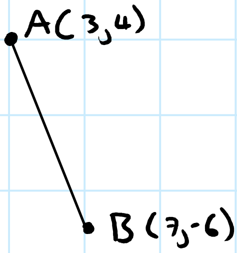
$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(4 - \sqrt{5})^2 = 4^2 + 2[4 \times -\sqrt{5}] + (-\sqrt{5})^2$$

$$= 16 - 8\sqrt{5} + 5$$

$$= 21 - 8\sqrt{5}$$

4) gradient of AB



$$\frac{\Delta y}{\Delta x} = \frac{-6-4}{7-3}$$

$$= \frac{-10}{4}$$

$$= -\frac{5}{2}$$

Perpendicular to AB

$\frac{2}{5}$ $\quad \because \frac{2}{5} \times -\frac{5}{2} = -1$

$$y - y_1 = m(x - x_1)$$

using $(3,4)$ as (x_1, y_1)

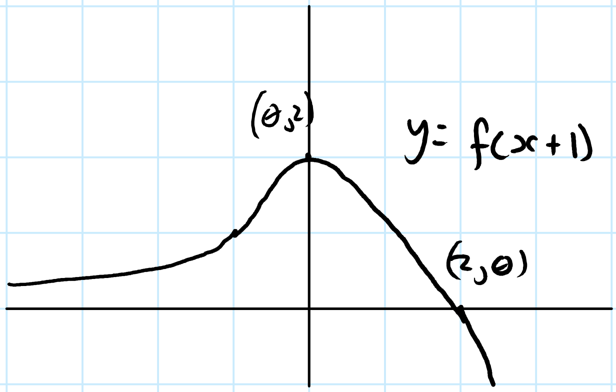
$$y - 4 = \frac{2}{5}(x-3)$$

$$5y - 20 = 2x - 6$$

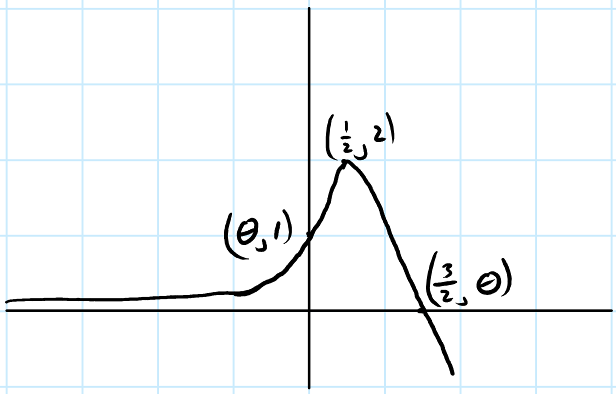
$5y - 2x - 14 = 0$ OR $2x - 5y + 14 = 0$

5)

a)



b)



6)

$y + 2x = 5$

$2x^2 - 3x - y = 16$

a)

$y = 5 - 2x$

$2x^2 - 3x - 5 + 2x = 16$

$2x^2 - x - 21 = 0$

$$\frac{1 \pm \sqrt{1^2 - -168}}{4} = \frac{1 \pm 13}{4}$$

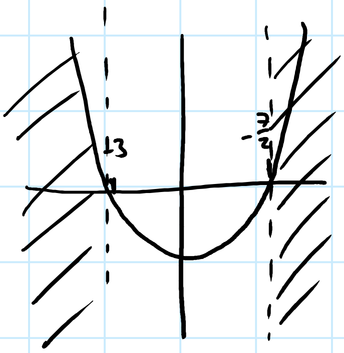
$x = -3, \frac{7}{2}$

$y = 5 - 2x$

$\therefore\ y = 11, -2$

b)

$2x^2 - x - 21 > 0$



$x < -3 \cup x > \frac{7}{2}$

Union, this means OR

7) $U_n = 250 + (n-1)50$, $a = 250$ $d = 50$

a) $U_{11} = 250 + 500$
$= £750$

b) $S_{20} = \frac{1}{2} 20 [2(250) + (20-1)50]$

$= 10[500 + 950]$

$= £14,500$

c) $U_n = A + (n-1)60$

$S_{20} = £14,500$

$14,500 = \frac{1}{2} 20 [2A + (20-1)60]$

$1,450 = 2A + 1,140$

$2A = 310$

$A = £155$

8) $x^2 + 10x + 36 = (x+a)^2 + b$

$\frac{10}{2} = 5$

$(x+5)^2 = x^2 + 10x + 25$

$x^2 + 10x + 36 = (x+5)^2 + 11$

b) $(x+5)^2 + 11 = 0$

$(x+5)^2 = -11$

$x + 5 = \pm\sqrt{-11}$

$x = \underbrace{\pm\sqrt{-11} - 5}$

these are not real roots

OR

$x^2 + 10x + 36$

Using the discriminant:

$b^2 - 4ac$

$10^2 - 4 \times 1 \times 36 = -44$

$-44 < 0 \therefore$

no real roots

c) $(1)x^2 + 10x + k = 0$

$b^2 - 4ac = 0$ Using discriminant

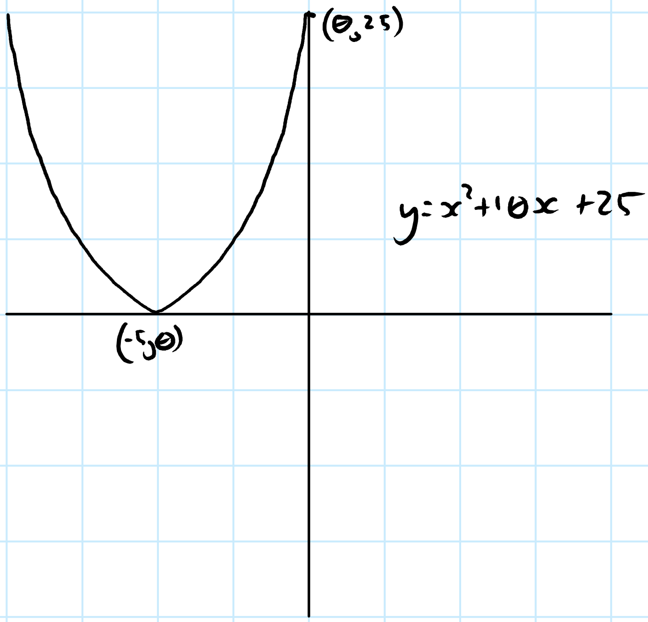
$10^2 - 4(1)k = 0$

$100 = 4k$

$k = 25$

8)

d)



9) $f'(x) = 3x^2 - 8x + 6$

a) $f(x) = \int f'(x)\,dx$

$= 3\frac{x^3}{3} - 8\frac{x^2}{2} + 6\frac{x^1}{1} + C$

$= x^3 - 4x^2 + 6x + C$

$f(3) = 27 - 36 + 18 + C$

$= 5$

$\therefore C = -4$

$f(x) = x^3 - 4x^2 + 6x - 4$

b) $f(2) = 8 - 16 + 12 - 4$

$= 0$

9)

c) $f'(3) = 3(9) - 8(3) + 6$

$= 9$

$3x^2 - 8x + 6 = 9$

$3x^2 - 8x - 3 = 0$

$\frac{8 \pm \sqrt{64 - -36}}{6} = \frac{8 \pm 10}{6}$

$x = 3, -\frac{1}{3}$

x co-ord of P is 3

$\therefore$ x co-ord of Q $= -\frac{1}{3}$

10)

a) $\frac{dy}{dx} = 3x^2 - 5 - 2x^{-2}$

A:

$\frac{dy}{dx}(1) = 3 - 5 - 2$

$= -4$

B:

$\frac{dy}{dx}(-1) = 3 - 5 - 2$

$= -4$

b) at A

$\frac{dy}{dx} = -4$

gradient of normal is $\frac{1}{4}$

$\therefore \frac{1}{4}x - 4 = -1$

$y - (-2) = \frac{1}{4}(x - 1)$

$4y + 8 = x - 1$

$4y = x - 9$

10) Since the gradients at A and B are equal, so too are the gradients of normal at A and B

c) gradient of Normal at B $= \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - -1)$ given $B(-1, 2)$

$4y - 8 = x + 1$

$4y = x + 9$

x Values of P & Q are 0 by definition

y value of P

$4y = 0 - 9$

$\therefore y = \frac{-9}{4}$

y value of Q

$4y = 0 + 9$

$y = \frac{9}{4}$

$\frac{9}{4} - -\frac{9}{4} = \frac{9}{2}$

$\overrightarrow{PQ} = \frac{9}{2}$